# Estimating the Optimal Parameter Values for BS－Horizon DEM Generation Algorithm in Flat Lowland Area 

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## 1．Introduction

This study aims to generate high resolution DEM of flat lowland area in Danang city，Vietnam for the purpose of flood inundation and flood hazard mapping． Topography plays an important role in flood modelling， hence it is necessary to create reliable DEM．An interpolation method based bi－cubic spline algorithm was used for generate 5 m resolution DEM．The generated DEM was used as input data for further extraction of geomorphology and flood risk zoning of this study area．

## 2．Methodology

## 2．1 Study area and data set

Study area comprises of $98 \mathrm{~km}^{2}$ of lowland area located in the south of Danang City with elevation ranging from 0 m to 10 m ．The data used for DEM generation are in－situ elevation points surveyed by Department of Natural Resource and Environment， Danang city in 2009，include 79,600 elevation points． This alluvial lowland area prone to experienced several serious flood events in past．

## 2．2．DEM generation process

In order to generate high resolution DEM of the study area，an equality－inequality constrained interpolation method implemented as BS－Horizon program（Nonogaki et al．，2012）was applied．The in－situ spot height data that includes 79，600 elevation points was taken into account as equality constraint to interpolate topographic surface．Further，the inequality constraint was characterized by the elevation range from 0 m to 10 m of the study area．In general，the possible constraint of an elevation $z_{i}$ that is obtained at a point $\left(x_{i}, y_{i}\right)$ can be expressed as follow：

$$
\begin{align*}
& f\left(x_{i}, y_{i}\right)-z_{i}=0  \tag{1}\\
& f\left(x_{i}, y_{i}\right)-z_{i}<0  \tag{2a}\\
& f\left(x_{i}, y_{i}\right)-z_{i}>0 \tag{2b}
\end{align*}
$$

Equality constraint（1）is used in case that the surface passes through the known elevation point．Inequality constraint（2a）is applied in cases that the surface passes under the point，and the inequality constraint（2b）is used in cases that the surface passes above the point
（Nonogaki et al．，2012）．
This equality－inequality constrained data was used as input data for BS－Horizon interpolation algorithm． BS－Horizon is a bi－cubic spline approximation that relies on an exterior penalty function to determine the optimal surface．Exterior penalty function controls the balance between the smoothness and the violation of the constraint using a penalty parameter（alpha）．The function can be shown in the equation 3：

$$
\begin{equation*}
Q(x, \alpha)=J(x)+\alpha R(x) \tag{3}
\end{equation*}
$$

Where $Q(x)$ is called an exterior penalty function，$J(x)$ evaluates the smoothness of the surface，$R(x)$ evaluates the degree of violation of constraints and $\alpha$ control a weight balance between $J(x)$ and $R(x)$ ．

In order to run BS－Horizon the equality－inequality constrained data needs to be prepared．Firstly，we colored the study area in a certain color ramp．A color table was prepared includes corresponding color ramp and the elevation constraint（ 0 m to 10 m ）．This colored map was saved as bitmap 24 bit image and imported to an application namely BMP2DAT．In this application the calculation region，the output cell size and the reference color table need to be specified．The output data is a text file of data points with long／lat location and inequality elevation constraints．Next，the spot height data was converted to equality constraint in the same format as the inequality data．Finally，both of these files were merged into an equality－inequality constraint data（in csv format）and input to BS－Horizon program．The process of BS－Horizon DEM generation can be described in Figure 1.

## 2．3 Estimating optimal parameters for BS－Horizon

According to BS－Horizon algorithm implemented by Nonogaki et al．（2012），it is observed that two main parameters effecting on the output DEM surface are $M$ （ $M_{x}$ or $M_{y}$ ，the number of sub－regions that constitute the surface domain，in which bi－cubic spline function works） and penalty parameter（alpha）．In order to obtain the best parameter values for BS－Horizon，batch processing was carried out by setting $J(x)$ and $R(x)$ values according to $M$ and $\alpha$ and examine the results are shown in a graph （Figure 2）．Three cases of $M_{x}$ and $M_{y}$ were used in this


Figure 1. DEM generation workflow.


Figure 2. Representation of $J(x)$ and $R(x)$ according to different $M$ and alpha.
experiment, namely: case 1: $M_{x}=189, M_{y}=123$; case 2: $M_{x}=643, M_{y}=438$ and case 3: $M_{x}=1072, M_{y}=730$. Alpha was used as values between 1 to $10^{10}$.

Figure 2 shows the transition tendency of $J(x)$ and $R(x)$ with the progress of iterative computation (Noumi, 2003). It can be observed that there are three different zones. Zone A shows the steady increase of $J$ and steady decrease of $R$, while in zone $\mathrm{B}, J$ increases continuously and $R$ goes to minimum values or the generated surface shows smaller errors. In zone C, $J$ continuously increases and $R$ shows spikes. It is suggested that B zone is the suitable for generating optimal surface. Several possible DEMs in this B zone were selected to calculate some statistical parameters pertaining to error estimation. Based on the given constraint with elevation range from 0 to 10 m and the root mean square error (RMSE) of DEMs, the optimal surface was selected (Table 1). Also, it is understandable to see from Figure 3 that DEM surface in case 3 shows more details of topography than the one in case 1.

## 3. Result

Considering the given inequality constraint, it is clear from Table 1 that the best parameters for BS-Horizon is $\alpha=2.34 \times 10^{7}, M_{x}=1072$ and $M_{y}=730$. As a result, the output DEM has elevation from $-0.3-10.06 \mathrm{~m}$, the RMSE is only 0.11 m . Compared to other interpolation methods, such as Inverse distance weighting (IDW) or Regularized spline with tension (RST), Horizon-DEM provides a better surface estimation and the vertical errors are also minimized as shown in (Table 2).

Table 1. Comparison of DEM with different parameter settings.

| Alpha | $M_{x}$ | $M_{y}$ | Min | Max | Mean | STD | RMSE |
| :---: | :---: | :---: | ---: | ---: | ---: | :--- | :---: |
| $2.07 \mathrm{E}+06$ | 189 | 123 | -1.9 | 13.5 | 3.05 | 1.98 | 0.40 |
| $6.95 \mathrm{E}+06$ | 189 | 123 | -3.3 | 16.1 | 3.06 | 2.0 |  |
| $2.34 \mathrm{E}+07$ | 189 | 123 | -15.4 | 31.5 | 3.07 | 2.1 |  |
| $2.07 \mathrm{E}+06$ | 643 | 438 | -0.7 | 10.6 | 3.03 | 1.99 | 0.16 |
| $6.95 \mathrm{E}+06$ | 643 | 438 | -2.0 | 10.9 | 3.03 | 2.0 |  |
| $2.34 \mathrm{E}+07$ | 643 | 438 | -2.5 | 10.6 | 3.03 | 2.0 |  |
| $2.07 \mathrm{E}+06$ | 1072 | 730 | -0.3 | 10.2 | 3.03 | 1.98 |  |
| $6.95 \mathrm{E}+06$ | 1072 | 730 | -0.4 | 10.3 | 3.03 | 1.99 |  |
| $2.34 \mathrm{E}+07$ | 1072 | 730 | -0.3 | 10.1 | 3.03 | 1.99 | 0.11 |

Table 2. BS-Horizon DEM compared with other methods.

| DEM | Min | Max | RMSE |
| :--- | :--- | :---: | ---: |
| BS-Horizon DEM | -0.3 | 10.1 | 0.11 |
| IDW DEM | $2.47 \times 10^{-9}$ | 10.6 | 0.29 |
| RST DEM | -9.0 | 10.3 | 0.21 |


(b) $M_{x}=1072, M_{y}=730$


Figure 3. DEM surface two case of $M_{x}, M_{y}$.

## 4. Conclusion

Field survey elevation point data combined with inequality constraint used in BS-Horizon interpolation method can provide reliable DEM. Experiment with different conditions $M_{x}, M_{y}$ and alpha parameter in BS-Horizon reveals that the most suitable for DEM generation for this flat lowland area are $M_{x}=1072, M_{y}=$ 730 and alpha $=2.34 \times 10^{7}$. Horizon DEM has better RMSE compare to other DEM interpolation methods such as IDW or RST.

## References

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